

Multiple Choice – Circle the correct answer – 3 points each – 30 points total

1. Which of the following factor(s) effect the shape of the yield curve:
 - I. the real return
 - II. interest rate risk
 - III. expected future inflation
 - IV. unexpected inflation
 - A. I only
 - B. I and II
 - C. I and III only
 - D. I, II and III
 - E. I, II, III and IV

2. The duration of a 10-year bond will _____ as interest rates decrease.
 - A. increase
 - B. decrease
 - C. remain the same
 - D. increase then decrease
 - E. Insufficient information.

3. The bond equivalent yield calculates interest on a _____ day year.
 - A. 360
 - B. 365
 - C. 366
 - D. 365 or 366
 - E. None of the above.

4. Which of the following is true?
 - A. The asked yield of a bond is always lower than the bid yield.
 - B. The asked yield of a bond is always higher than the bid yield.
 - C. It is possible for the bid price of a bond to be higher than the asked price.
 - D. There is no definitive relationship between bid-ask yields and bid-ask prices.
 - E. None of the above are true.

5. A Treasury bill with a face value of \$10 million has an asked price of \$9,727,000 and a discount yield of 6.51%. How many days until this bill matures?
- A. 151 days
 - B. 156 days
 - C. 160 days
 - D. 165 days
 - E. 169 days
6. Convertible bonds:
- A. give bondholders the ability to share in the price appreciation of the stock.
 - B. offer a lower coupon rate than similar nonconvertible bonds.
 - C. offer a higher coupon rate than similar nonconvertible bonds.
 - D. both a and b.
 - E. both b and c.
7. Which of the following theories states the shape of the yield curve is essentially determined by the supply and demand for short- and long-maturity bonds?
- A. Market segmentation.
 - B. Rational expectations.
 - C. Modern theory.
 - D. Liquidity preference.
 - E. None of the above.
8. Rank BBB corporate bonds, U.S. Treasury bonds, U.S. government agency debt and AAA corporate bonds based on their yield from lowest to highest.
- A. U.S. Treasury bonds, U.S. government agency bonds, BBB corporate bonds, AAA corporate bonds
 - B. BBB corporate bonds, U.S. Treasury bonds, U.S. government agency bonds, AAA corporate bonds
 - C. U.S. Treasury bonds, AAA corporate bonds, BBB corporate bonds, U.S. government agency bonds
 - D. U.S. Treasury bonds, AAA corporate bonds, U.S. government agency bonds, BBB corporate bonds
 - E. U.S. Treasury bonds, U.S. government agency bonds, AAA corporate bonds, BBB corporate bonds
9. Assume the market segmentation theory is true. If investors believe that inflation will be higher in the future than it is now, then the term structure of interest rates will be:
- A. upward sloping.
 - B. flat.
 - C. downward sloping.
 - D. humped.
 - E. Any of the above.

10. Which of the following bonds will have the smallest price change from the same change in interest rates, all else the same?
- A. 5-year, 10 percent coupon.
 - B. 5-year, 6 percent coupon.
 - C. 10-year, 10 percent coupon.
 - D. 10-year, 6 percent coupon.
 - E. 10-year, zero coupon.

Partial Credit Problems – Show All Work – 70 Total Points

Problem 1 (15 points) You have a bond with 24 years to maturity, a semiannual coupon rate of 10 percent and a market interest rate of 8 percent. Using duration and convexity (Macaulay, modified and effective), what is the expected change in the bond's price if the YTM rises by 100 basis points. Compare your answer with the price you calculate for the bond's new price.

Problem 2 (5 points) List and explain five factors specific to a bond that could affect the coupon rate (or yield to maturity) of a corporate bond.

Problem 3 (5 points) There are six well-known theorems regarding the relationship between interest rates and bond prices. List and explain five of them.

Problem 4 (15 points) You are responsible for managing a corporate liability fund. The fund is expected to make the payments below each year for the next five years and then make payments of \$40 million per year for the following 25 years.

Year	Cash flow
1	\$10,000,000
2	\$18,000,000
3	\$25,000,000
4	\$29,000,000
5	\$35,000,000

Assume all cash flows will occur at the end of the year and the interest rate is 5 percent. You have two bond issues that can be used to immunize our portfolio. Your company already owns \$200 million worth of Bond Y with a 6.5-year maturity bond with a duration of 5.4 years. You will keep this bond investment to immunize your liability. What is the required duration of the second bond necessary to fully immunize the liability?

Problem 5 (10 points) Assume you find the following quotes on Treasury STRIPS. What is the one-year forward rate in three years? The one-year forward rate in four years? What price would you expect to pay for a two-year STRIPS on March 29, 2020?

U.S. Treasury STRIPS, March 29, 2017

Maturity	Price
Feb 20	90.221
Feb 21	84.498
Feb 22	80.093

Problem 6 (10 points) You are given the following information on four coupon bonds. All coupons are paid annually.

<u>Bond</u>	<u>Coupon</u>	<u>Time to Maturity</u>	<u>Yield to Maturity</u>
A	7%	1 year	5%
B	6%	1 year	5%
C	7.5%	2 years	6.2%
D	8%	3 years	7.3%

What are the one year, two year, and three year zero coupon bonds rates?

Problem 7 (10 points) Rank the following bonds in order of descending (longest to shortest) duration. Explain how you ranked the bonds. If the duration relationship between two or more bonds cannot be determined, explain why.

<u>Bond</u>	<u>Coupon</u>	<u>Time to Maturity</u>	<u>Yield to Maturity</u>
A	15%	20 years	R%
B	15%	15 years	R%
C	0%	20 years	R%
D	8%	20 years	R%
E	15%	15 years	R + 1%

Multiple Choice

- | | |
|------|-------|
| 1. D | 6. D |
| 2. A | 7. A |
| 3. D | 8. E |
| 4. A | 9. E |
| 5. A | 10. A |

Problem #1

Maturity	24
Coupon	10.00%
YTM	8.00%
Interest rate change	1.00%

Macaulay duration 21.228025 half years
 years

P ₀	\$1,211.951308814
P ₊	\$1,210.715420818
P ₋	\$1,213.189204144

Convexity

Modified duration

V ₀	\$1,211.95
V ₋	\$1,346.37
V ₊	\$1,097.68

Effective duration

Effective convexity

Actual new price

New price with Macaulay duration and convexity

Price change	\$ (113.65)
New price	\$ 1,098.30

New price with modified duration and convexity

Price change	\$ (114.53)
New price	\$ 1,097.42

Problem #2 A possible list of factors include bond rating, maturity, seniority, sinking fund, callability, putability, protective covenants (or lack of), and the liquidity of the bond.

Problem #3

1. There is an inverse relationship between interest rates and bond prices. If interest rates increase, bond prices decrease. If interest rates decrease, bond prices increase.
2. An increase in a bond's yield to maturity results in a smaller price change than a price decrease from an increase in the yield to maturity of the same magnitude.
3. Interest rate risk is the risk that if interest rates increase, bond prices will decrease. All else the same, a longer term bond will have more interest rate risk than a shorter term bond.
4. The sensitivity of bond prices to changes in yields increases at a decreasing rate as maturity increases. In other words, interest rate risk is less than proportional to maturity.
5. All else the same, there is an inverse relationship between the coupon rate and interest rate risk. A bond with a lower coupon has more interest rate risk than a bond with a higher coupon.
6. The sensitivity of a bond's price to a change in its yield is inversely related to the yield to maturity at which the bond is currently selling. A low YTM will result in a greater interest rate sensitivity while a high YTM will result in a lower interest rate sensitivity.

Problem #4

The duration of the liability is:

Year	Cash flow	PV	PV x T
1	\$10,000,000	\$9,523,809.52	\$9,523,809.52
2	\$18,000,000	\$16,326,530.61	\$32,653,061.22
3	\$25,000,000	\$21,595,939.96	\$64,787,819.89
4	\$29,000,000	\$23,858,371.77	\$95,433,487.08
5	\$35,000,000	\$27,423,415.83	\$137,117,079.13
		<hr/>	<hr/>
		\$98,728,067.69	\$339,515,256.85

$$D = \$339,515,256.85 / \$98,728,067.69 = 3.43889 \text{ years}$$

$$\text{Value of annuity in 5 years} = \$563,757,782.64$$

$$\text{Value of annuity today} = \$441,718,974.25$$

$$\text{Duration of annuity} = 5 + [(1 + .05) / .05] / \{25 / [(1 + .05)^{25} - 1]\} = 10.52377$$

$$\text{Value of liability today} = \$98,728,067.69 + 441,718,974.25 = \$540,447,041.94$$

$$\begin{aligned} \text{Duration of liability} &= \$98,728,067.69 / \$540,447,041.94(3.43889) \\ &\quad + \$441,718,974.25 / \$540,447,041.94(10.52377) \end{aligned}$$

$$\text{Duration of liability} = 9.2295 \text{ years}$$

$$\text{Weight of Y} = \$200,000,000 / \$540,447,974.25 = .3701$$

$$\text{Weight of Z} = 1 - .3701 = .6299$$

$$\begin{aligned} w_Y D_Y + w_Z D_Z &= 9.2295 \text{ years} = .3701(5.4) + .6299 D_Z \\ .6299 D_Z &= 7.23312 \\ D_Z &= 11.4792 \end{aligned}$$

Problem #5

Spot rates:

$$3\text{-year rate: } 90.221 = 100/(1 + R)^6 \Rightarrow R = 0.01730 \text{ or } 3.46\%$$

$$4\text{-year rate: } 84.498 = 100/(1 + R)^8 \Rightarrow R = 0.02128 \text{ or } 4.26\%$$

$$5\text{-year rate: } 80.093 = 100/(1 + R)^{10} \Rightarrow R = 0.02245 \text{ or } 4.49\%$$

One-year rate in 3 years:

$$1.02128^8 = (1.01730)^6(1 + f_{1,3})^2$$

$$\left(\frac{(1.02128)^8}{(1.01730)^6} \right)^{1/2} - 1 = .03331$$

$$f_{1,3} = .0331 \times 2 = 6.662\%$$

One-year rate in 4 years:

$$1.02245^{10} = (1.02128)^8(1 + f_{1,4})^2$$

$$\left(\frac{(1.02245)^{10}}{(1.02128)^8} \right)^{1/2} - 1 = .02713$$

$$f_{1,3} = .02713 \times 2 = 5.426\%$$

Two-year rate in 3 years:

$$1.02445^{10} = (1.01730)^6(1 + f_{2,3})^4$$

$$\left(\frac{(1.02445)^{10}}{(1.01730)^6} \right)^{1/4} - 1 = .03022$$

$$f_{2,3} = .03022 \times 2 = 6.043\%$$

2-year STRIPS in three years:

$$\text{Price} = 100/(1 + .03022)^4 = 88.774$$

Problem #6

Bond prices:

<u>Bond</u>	<u>Price</u>
A	\$1,019.05
B	\$1,009.52
C	\$1,023.77
D	\$1,018.27

The one year rate must be 5% since the two coupon bonds are essentially zero coupon bonds. The two year rate is:

$$\$1,023.77 = \$75 / 1.05 + \$1,075/(1 + R_2)^2$$

$$\$952.34 = \$1,075/(1 + R_2)^2$$

$$(1 + R_2)^2 = \$1,075 / \$952.34$$

$$R_2 = 6.24\%$$

And the 3 year rate is:

$$\$1,018.27 = \$80 / 1.05 + \$80/(1.0624)^2 + \$1,080/(1 + R_3)^3$$

$$\$871.20 = \$1,080/(1 + R_3)^3$$

$$(1 + R_3)^3 = \$1,080 / \$871.20$$

$$R_2 = 7.42\%$$

Problem #7

Duration is affected by the maturity of the bond, coupon rate, and the yield to maturity. Of these three factors, maturity is the most influential. Since the bonds have maturities of 15 and 20 years, we can say that the 20-year bonds will have a higher duration, given that the 15-year bonds are coupon bonds. Therefore we evaluate the 20-year bonds first. The bonds all have the same yield to maturity. We also know that coupon rates and duration are inversely related. We can therefore order these bonds by coupon rate, from lowest to highest. The two 15 year bonds have the same maturity and coupon rate, but different yield to maturities. All else constant, the duration of a coupon bond is higher when the yield to maturity is lower, therefore B has a higher duration than E. The bond duration, from highest to lowest, is:

<u>Bond</u>	<u>Coupon</u>	<u>Time to Maturity</u>	<u>Yield to Maturity</u>
C	0%	20 years	10%
D	8%	20 years	10%
A	15%	20 years	10%
B	15%	15 years	10%
E	15%	15 years	15%