Put-Call Parity with Known Dividend
$C - P = S - (Div)e^{-Rt} - Xe^{-Rt}$

Put-Call Parity with Continuous Dividends
$P = C + Xe^{-Rt} - S_0 e^{-yt}$

Black-Scholes-Merton Model
$C_0 = S_0 e^{-yt} N(d_1) - Xe^{-Rt} N(d_2)$
$P_0 = Xe^{-Rt} N(-d_2) - S_0 e^{-yt} N(-d_1)$

$$d_1 = \frac{ln\left(\frac{S}{X}\right) + \left(R - y + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$d_2 = d_1 - \sigma\sqrt{t}$

Delta of a call = $e^{-yt} N(d_1)$
Delta of a put = $-e^{-yt} N(-d_1)$

Eta of a call = $e^{-yt} N(d_1)(S/C)$
Eta of a put = $-e^{-yt} N(-d_1)(S/P) < 0$

Vega = $S_0 e^{-yt} N'(d_1)\sqrt{t}$

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{x^2}{2}\right)}$$

$$\text{Gamma} = \frac{N'(d_1)e^{-yt}}{S_0\sigma\sqrt{t}}$$

$$\text{Call theta} = -\frac{S_0 N'(d_1)\sigma e^{-yt}}{2\sqrt{t}} + yS_0 N(d_1)e^{-yt} - RXe^{-Rt}N(d_2)$$

$$\text{Put theta} = -\frac{S_0 N'(d_1)\sigma e^{-yt}}{2\sqrt{t}} - yS_0 N(-d_1)e^{-yt} + RXe^{-Rt}N(-d_2)$$

Call rho = $Xte^{-Rt} N(d_2)$
Put rho = $-Xte^{-Rt} N(-d_2)$

Hedging with index options

$$\text{Number of option contracts} = \frac{\text{Portfolio beta} \times \text{Portfolio value}}{\text{Option delta} \times \text{option contract value}}$$

Binomial trees

$$p^* = \frac{e^{\mu\Delta t} - d}{u - d}$$

$u = e^{\sigma\sqrt{\Delta t}}$
$d = e^{-\sigma\sqrt{\Delta t}} = 1/u$
$f = e^{-R\Delta t}[pf_u + (1 - p)f_d]$

Known Dividend

$S^* = S_0 - (\text{Dividend})e^{-Rt}$

Continuous dividend yield and binomial trees

$p = \frac{e^{(R-q)\Delta t} - d}{u-d}$

$u = e^{\sigma\sqrt{\Delta t}}$

$d = e^{-\sigma\sqrt{\Delta t}} = 1/u$

Options on futures

$u = e^{\sigma\sqrt{\Delta t}}$

$d = e^{-\sigma\sqrt{\Delta t}} = 1/u$

$p = \frac{1-d}{u-d}$

Money Markets

$\text{Price} = \text{Face value}\left(1 - \frac{\text{Days}}{360} \times R_{BD}\right)$

$R_{BD} = \frac{\text{Par-Price}}{\text{Par}} \times \frac{360}{n}$

$R_{BEY} = \frac{\text{Par-Price}}{\text{Price}} \times \frac{365}{n}$

$R_{BEY} = \frac{365 \times R_{BD}}{360 - (R_{BD} \times n)}$

$\text{Equivalent taxable yield} = \frac{\text{Tax-exempt yield}}{\text{1-Marginal tax rate}}$

$\text{Critical tax rate} = 1 - \frac{R_M}{R}$

Accrued interest

30/360

If D1 = 31, change to 30

If D2 = 31 and D1 = 30 or 31, change D2 to 30, otherwise leave D2 at 31

# of days

(Y2 – Y1)×360 + (M2 – M1)×30 + (D2 – D1)

30E/360 – Assumes a 30-day month

If D1 = 31, change to 30

If D2 = 31 Change to 30

# of days

(Y2 – Y1)×360 + (M2 – M1)×30 + (D2 – D1)

$w = \frac{\text{\# of days between settlement and next coupon payments}}{\text{\# of days in coupon period}}$

$$\text{Accrued interest} = C\left(\frac{\text{\# of days since last coupon}}{\text{\# of days in period}}\right)$$

Duration and Convexity

$$\frac{\partial P}{P} = -D\left(\frac{\partial R}{1+R}\right)$$

$$\frac{\partial P}{P} = -D\left(\frac{\partial R}{1+(R/2)}\right)$$

$$D = \frac{\sum DCF \times t}{\sum DCF\ (\text{price})}$$

$$D = \frac{1+y}{y} - \frac{(1+y)+T(c-y)}{c[(1+y)^T - 1] + y}$$

Duration of a perpetuity is: $\frac{1+y}{y}$

Duration for a level annuity is: $\frac{1+y}{y} - \frac{T}{(1+y)^T - 1}$

$$\partial P = P \times [(-D) \times \left[\frac{\partial R}{1+R}\right]]$$

$$\frac{\partial P}{P} = -D\left[\frac{\Delta R}{1+R}\right] + \frac{1}{2}CX(\Delta R)^2$$

CX = convexity = Scaling factor [capital loss from one basis point rise in R + capital gain from one basis point drop in R]

$$D_M = \frac{D}{1+y}$$

%Δ in bond price = $-D_M(\Delta R)$

$$D_E = \frac{V_- - V_+}{2V_0(\Delta R)}$$

V_0 = initial price
V_- = price if YTM decreases by R
V_+ = price if YTM increases by R

$$CX_E = \frac{V_- + V_+ - 2V_0}{2V_0(\Delta R)^2}$$

Futures
$F_T = S(1 + R - d)^T$

Stock hedging with futures

$$\text{\# of contracts} = \frac{\beta_P \times V_P}{V_F}$$

Bond hedging with futures

$$\text{\# of contracts} = \frac{D_P \times V_P}{D_F \times V_F}$$

Cross Hedging

$$h = \rho_{S,F}\left(\frac{\sigma_S}{\sigma_F}\right)$$

Value at Risk

Portfolio variance for 2 asset portfolio (total risk) = $w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B Cov(A,B)$
Portfolio variance for 2 asset portfolio (total risk) = $w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \sigma_A \sigma_B \rho_{A,B}$

$E(R_{P,T}) = E(R_P) \times T$
$\sigma_{P,T} = \sigma_P \times \sqrt{T}$

$Prob[R_{P,T} \leq E(Rp) \times T - 2.326\sigma_P\sqrt{T}\ ] = 1\%$
$Prob[R_{P,T} \leq E(Rp) \times T - 1.96\sigma_P\sqrt{T}\ ] = 2.5\%$
$Prob[R_{P,T} \leq E(Rp) \times T - 1.645\sigma_P\sqrt{T}\ ] = 5\%$