Put-Call Parity
$P = C + Xe^{-Rt} - S_0e^{-yt}$

Black-Scholes-Merton Model
$C_0 = S_0e^{-yt}N(d_1) - Xe^{-Rt}N(d_2)$
$P_0 = Xe^{-Rt}N(-d_2) - S_0e^{-yt}N(-d_1)$

$$d_1 = \frac{ln\left(\frac{S}{X}\right) + \left(R - y + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$d_2 = d_1 - \sigma\sqrt{t}$

Delta of a call = $e^{-yt}N(d_1)$
Delta of a put = $-e^{-yt}N(-d_1)$

Eta of a call = $e^{-yt}N(d_1)(S/C)$
Eta of a put = $-e^{-yt}N(-d_1)(S/P) < 0$

Vega = $S_0e^{-yt}N'(d_1)\sqrt{t}$

$$N'(x) = \frac{1}{\sqrt{2\pi}}e^{-(\frac{x^2}{2})}$$

$$\text{Gamma} = \frac{N'(d_1)e^{-yt}}{S_0\sigma\sqrt{t}}$$

$$\text{Call theta} = -\frac{S_0N'(d_1)\sigma e^{-yt}}{2\sqrt{t}} + yS_0N(d_1)e^{-yt} - RXe^{-Rt}N(d_2)$$

$$\text{Put theta} = -\frac{S_0N'(d_1)\sigma e^{-yt}}{2\sqrt{t}} - yS_0N(-d_1)e^{-yt} + RXe^{-Rt}N(-d_2)$$

Call rho = $Xte^{-Rt}N(d_2)$
Put rho = $-Xte^{-Rt}N(-d_2)$

Hedging with index options

$$\text{Number of option contracts} = \frac{\text{Portfolio beta} \times \text{Portfolio value}}{\text{Option delta} \times \text{option contract value}}$$

Binomial trees

$$p^* = \frac{e^{\mu\Delta t} - d}{u - d}$$

$u = e^{\sigma\sqrt{\Delta t}}$
$d = e^{-\sigma\sqrt{\Delta t}} = 1/u$
$f = e^{-R\Delta t}[pf_u + (1 - p)f_d]$

Known Dividend
$S^* = S_0 - (\text{Dividend})e^{-Rt}$

Continuous dividend yield and binomial trees

p = $\frac{e^{(R-q)\Delta t} - d}{u-d}$

u = $e^{\sigma\sqrt{\Delta t}}$

d = $e^{-\sigma\sqrt{\Delta t}}$ = 1/ u

Options on futures

u = $e^{\sigma\sqrt{\Delta t}}$

d = $e^{-\sigma\sqrt{\Delta t}}$ = 1/ u

p = $\frac{1 - d}{u-d}$