Black-Scholes-Merton Option Pricing Model (Alternate Version with dividends)

S = Stock price
X = Strike price
t = Time to maturity
r = Risk-free rate
De^{-rt} = Present value of all dividends over the life of the option
σ = Stock price volatility (standard deviation)

$$C = (S - De^{-rt})N(d_1) - Xe^{-rt}N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S-De^{-rt}}{X}\right)+\left(r-y+\frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

Put-Call Parity

$$P = C + Xe^{-rt} - (S - De^{-rt})$$

Ignore dividends:

$$P = C + Xe^{-rt} - S$$

We can re-write as:

$$S = C + Xe^{-rt} - P$$

A stock can be replicated by a long call (to capture the upside gains), a short put (to reflect the downside losses), and a T-bill (to capture the time-value component–the "wait" factor).

Implied Volatility

Volatility is the only unmeasurable variable in Black-Scholes

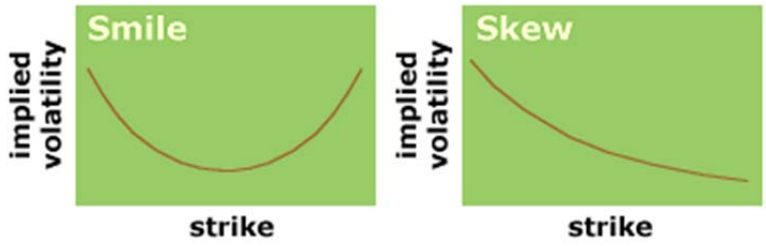
Implied Volatility for an option with a stock price near the strike price:

$$\sigma \approx \frac{\sqrt{2\pi/t}}{A+B}\left[C - \frac{A-B}{2} + \sqrt{\left(C - \frac{A-B}{2}\right)^2 - \frac{(A-B)^2}{\pi}}\right]$$

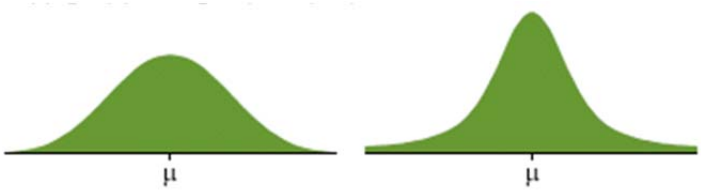
A = Se^{-yt}
B = Xe^{-rt}

OR: www.numa.com

Volatility Skews and Smiles



Why?
1) Non-normality of returns. Actual returns are more leptokurtic. Market leptokurtosis would make way out-of-the-money or way in-the-money options more expensive than would be assumed by the Black-Scholes formulation. By increasing prices for such options, a volatility smile could be the markets' indirect way of achieving such higher prices within the imperfect framework of the Black-Scholes model.



These graphs illustrate the notion of kurtosis. The PDF on the right has higher kurtosis than the PDF on the left. It is more peaked at the center, and it has fatter tails.

2) Market fear of crashes. The fear of crashes could cause investors to bid up prices of options at strike prices below the current market price.

3) Constant volatility assumption of Black-Scholes.

Questions

General Motors: Valuation of Class E Contingent Notes

1. Is the contingent note a put or a call? Why?

2. What are the appropriate parameters to use in estimating the option's value? Defend your choice of time to maturity, risk-free rate, strike price, expected dividends, and volatility?

3. Using the dividend-adjusted Black-Scholes model, what is your estimate of the notes' total liability to General Motors? Why did you use the particular dividend adjustment to Black-Scholes that you did?

4. What advantages did GM gain by using the notes as part of the compensation package for Electronic Data Systems' shares? Can you think of any other compensation package that could have offered a similar payoff to EDS shareholders?