Do Long Interest Rates Ever Fall?

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ABSTRACT

Theoretical considerations in Dybvig, Ingersoll, and Ross (1996) lead the authors to conclude that long forward and zero-coupon rates can never fall. We examine this conjecture empirically using monthly U.S. Treasury STRIPS data over the period 1990 – 2000. Based on the Cox, Ingersoll, and Ross (1985) term structure model and a constant-drift adaptation of that model, we find that, contrary to predictions, implied long maturity zero-coupon rates did fall substantially during the last half of this period.
I. INTRODUCTION

Do long interest rates ever fall? On the face of it, the question might seem a little absurd. Casual inspection of, for example, long-term U.S. Treasury bond yields shows that they declined from a high of 14% in 1980 to under 5% in early 2006. Nonetheless, despite the historical record of long-term yields, theoretical analysis in Dybvig, Ingersoll, and Ross (DIR, 1996) leads to the prediction that interest rates for zero-coupon default-free bonds with maturities substantially longer than are currently available should never fall more than trivially.

Our primary goal in this paper is to empirically examine the behavior of long-term zero-coupon rates. A fundamental problem for such research is that rates for maturities much longer than 30 years are unobservable. We therefore estimate a commonly-used term structure model using monthly data on U.S. Treasury coupon STRIPS covering the period 1990 – 2000 and examine the long maturity zero-coupon rates implied by the fitted term structures. To our knowledge, we are the first to extensively investigate the empirical behavior of such rates. Our primary finding is that long interest rates are relatively stable over the first half of the 1990s; however, contrary to the theoretical analyses in DIR, these rates fall substantially over the last half of the decade.

The remainder of the paper is organized as follows. The next section reviews the arguments regarding long forward and zero-coupon rates. Section III
describes our data and methods. Section IV presents the empirical results and
Section V concludes the paper.

II. WHY LONG RATES SHOULD NEVER FALL

The essence of Dybvig, Ingersoll, and Ross’s (DIR, 1996) argument is that
if long (i.e., infinite maturity) forward and zero-coupon rates can decline, then,
asymptotic arbitrage opportunities exist. Berk (1991) extends the analysis by
showing how to construct the relevant trade and by illustrating that rates for
sufficiently long, but finite maturities cannot fall more than trivially. Berk also
provides conditions under which long rates must be constant and can thus neither
rise nor fall. Hubalek, Klein, and Teichmann (2002) further extend the analysis by
showing that the ergodicity assumption of DIR is not needed to prove that long-
term rates must remain constant. Dybvig and Marshall (1996) provide some
additional insights in the context of yields on long-maturing coupon bonds.

To illustrate why long rates should never fall, we take an approach that
differs somewhat from that in DIR. Our goal is to illustrate that very standard
bond pricing formulations also imply that long rates should not fall. Compared to
DIR, we sacrifice some generality and mathematical elegance in exchange for a
clear and simple economic intuition, regarding the behavior of long rates.

A Simple Example

It is useful to begin with a simple and very artificial example. However, as
we illustrate below, this example can be extended quite easily to illustrate more
general results. Suppose that in one year all interest rates for all maturities will be either 6 percent or 8 percent and stay that way forever. The expected price of a $1 face value zero-coupon bond in one year is \( e^{-0.06(T-t-1)} \) with probability \( p \) and priced at \( e^{-0.08(T-t-1)} \) with probability \((1 - p)\), where \( T - t - 1 \) is the time to maturity in one year (\( T \) is the final maturity date; \( t \) is the current date). In other words, beginning next year, the yield curve will be completely flat and interest rate uncertainty will vanish forever. The only uncertainty today is whether yields will be 6 or 8 percent. Under these circumstances, what does the yield curve look like today?

It is clear that the expected yields in one year are the same at all maturities. This might lead to the belief that the yield curve would be flat at some level today, but this is not the case. To demonstrate this, consider the price today (at time \( t \)) of a zero-coupon bond with a $1 face value that matures at time \( T \), \( P(t,T) \):

\[
P(t,T) = e^{-y(t,1)} E_t[P(t+1,T)]
\]

\[
= e^{-y(t,1)} \left[ pe^{-0.06(T-t-1)} + (1 - p)e^{-0.08(T-t-1)} \right] \tag{1}
\]

\[
= pe^{-y(t,1)} e^{-0.06(T-t-1)} \left[ 1 + \frac{1-p}{p} e^{-0.08-0.06(T-t-1)} \right]
\]

In equation (1), \( y(t,1) \) is the current one-period rate, and \( E_t[P(t+1,T)] \) is the expected price next period based on the information available today.

The continuously compounded yield to maturity, \( y(t,T) \), on this bond is:
Equation (2) shows that the yield for a bond with a finite maturity depends on the one-period rate, \( y(t,1) \), the probability, \( p \), as well as time to maturity, \( T-t \).

However, as time to maturity grows, the yield approaches 6 percent and reaches it in the limit:

\[
\lim_{{T \to \infty}} y(t,T) = 0.06 - 0 + 0 - 0 - 0 = 0.06
\]

As equation (3) shows, the long rate must be equal to 6 percent today. Because 6 percent is the lowest possible rate next period, the long rate cannot fall.

To illustrate the shape of the term structure in equation (2), we assume that the one-period rate, \( y(t,1) \), is equal to 7 percent. Figure I illustrates how the term structure appears for some different values of probability, \( p \). As shown, for smaller values of \( p \), the term structure is humped, but, as long as the probability of a rate fall is not zero, the term structure will, at some point, begin to asymptotically approach 6 percent. Furthermore, it is not necessary to constrain the current one-period rate to be between 6 and 8 percent; the only requirement is that it must be finite.
The reason that interest rates must eventually approach 6 percent in our example has to do with a familiar effect in bond valuation, namely, convexity. It is well known that interest rate increases and decreases have an asymmetric effect on bond prices, and, further, the asymmetry becomes more pronounced as maturity grows. To illustrate this, Figure II shows the percentage change in the price of a pure discount bond from a one basis point change in interest rates (i.e., the value of an ’01). The potential loss on the long discount bond is limited by the purchase price, but the potential gain becomes greater and greater as maturity increases. For example, the price of a 25,000 year maturity zero-coupon bond will fall by about 92 percent for a 1 basis point increase in interest rates. However, for a 1 basis point decrease in interest rates, the bond’s price will increase in value by over 1,100 percent.

These values imply that the probability of a decrease in interest rates must be very small. To see this, suppose that the only possible change in interest rates was 1 basis point in either direction in the next, say, week or month. If the probabilities were 50-50, then the expected return on the 25,000-year bond in our example would be .50(1,100%) + .50(–92%) = 504%. Thus, for the expected return to be economically sensible (and to avoid arbitrage possibilities), there must be very little chance of even a very small drop in rates.
The fact that long rates should not fall has important implications for term structure theory. In particular, it places a constraint on term structure models; any model that implies that long interest rates can decline, is not arbitrage-free. It also has significant practical implications for the valuation of very long-term coupon bonds, such as the “century” bonds floated in the last decade by a number of issuers.

III. DATA AND EMPIRICAL METHODS

Data

Our data are monthly observations on bid prices for U.S. Treasury coupon STRIPS over the full decade plus period January 1, 1990 through December 31, 2000. The data are from Street Software, Inc., a commercial data vendor; the underlying source for the data is Bear Stearns. The same source is used for quotes that appear in, e.g., The Wall Street Journal. The prices are trader bid quotes as of mid-afternoon. We use only coupon STRIPS because of their generic nature relative to principal STRIPS.¹

STRIPS have a number of characteristics that make them particularly amenable to fitting term structure models. First, since these securities are default-free, pure discount bonds, and are actively traded, there is no need to indirectly infer the zero-coupon yield curve from coupon bond data, thereby eliminating a potential source of noise. Second, STRIPS are taxed homogeneously, while

¹ See Jordan, Jorgensen, and Kuipers (2000) for a detailed analysis of the STRIPS prices, including a comparison of the relative prices of principal and coupon STRIPS.
coupon-bearing Treasury instruments are taxed asymmetrically (at least for some investors) according to premium or discount status. Thus, the tax effects documented in previous studies should be absent. Finally, STRIPS exist at regularly-spaced quarterly maturity intervals whereas cash Treasury instruments are heavily concentrated at short maturities, and there is a “hole” in the useable yield curve over the period we study extending from the years 2006 through 2014 because of callable issues. The equal maturity spacing for STRIPS is particularly useful, because, depending on the estimation method and how it is employed, term structure estimates using cash market Treasury securities place unequal weights on particular maturity ranges.

**Methods**

The very long-term interest rates that are the subject of DIR’s theoretical work are unobservable; consequently, we estimate these rates using the well-known Cox, Ingersoll, and Ross (CIR, 1985) model, along with a constant-drift adaptation of the model. As is well known, these models have the property that the long forward and zero-coupon rates are constant. These models can be viewed as special cases of the time-homogenous, one-factor, affine class of term structure models.

To investigate the appropriateness of using a single factor model, we perform a maximum likelihood factor analysis of STRIPS yields. As shown in Table 1, the first common factor accounts for over 97 percent of the variation in
the STRIPS yield curve, while the second factor explains virtually everything else. While there is some residual variance not explained by a single common factor, this is primarily at the short end of the term structure and our focus is on long rates.

The basic model we estimate is the CIR model, which assumes that the instantaneous short rate evolves according to:

$$dr = \kappa(\mu - r)dt + \sigma \sqrt{r}dz$$  \hspace{1cm} (4)

where $dr$ is the change in the short interest rate $r$ over a small interval of time, $\kappa$ is the speed of adjustment coefficient, $\mu$ is the stationary point, and $\sigma$ is the volatility term.

The most direct method of estimating these models, pioneered by Brown and Dybvig (1986), is to fit the relevant equations to a cross-section of bills, notes, and bonds. As several authors have noted, the cross-sectional procedure is similar to the computation of implied volatilities in the Black-Scholes option pricing model; observed prices are “inverted” to recover unknown parameters. In the current context, the primary advantage of the cross-sectional approach is that it fully incorporates all market information regarding the term structure in that instruments of various maturities are included in the sample. In addition,

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2 Similar results are found in Jordan and Kuipers (2002).
measurement errors and noise in the data become part of the statistical method rather than serving to potentially bias the estimated parameters.

We fit the CIR model and a constant-drift adaptation of the CIR model to the STRIPS data each month. The constant-drift adaptation of the CIR model is:

\[ dr = \mu dt + \sigma \sqrt{r} dz \]  \hspace{1cm} (5)

Note that in this simplified model, \( \mu \) is the constant expected drift in interest rates rather than the stationary point. As we illustrate below, this simpler model can sometimes be successfully estimated when the full model cannot be due to convergence problems. The result is two time series of estimated long rates, each containing 132 observations (depending on convergence). Two aspects of the estimation merit comment.

First, with the STRIPS data, we have the choice of estimating either the discount function (using STRIPS prices) or the term structure (using the STRIPS continuously compounded yields). In contrast, with coupon-bearing instruments, only the discount function can be estimated initially. Of course, if the data were measured without error and the models were fully correct specifications, the choice would make no difference, but, once we allow for specification and data errors, matters are not so simple.

To investigate this issue, we compare two specifications. In the first, we assume observed STRIPS prices are equal to true model prices plus a homoskedastic error. The second specification makes the same assumption for
observed yields. Note that, because of the nonlinear relation between prices and yields, both specifications cannot be correct. For example, homoskedastic errors in yields imply multiplicative, heteroskedastic errors in prices (they also imply that prices are upward biased because of Jensen’s inequality). Previous analysis by Jordan and Kuipers (2002) show that the yield specification is superior as it produces estimates for the underlying rate process that more closely correspond to actual observed interest rates and their behavior.

In our estimations, we only use data out to 25 years because of concerns raised by DIR regarding coupon STRIPS at the long end of the yield curve. Further support for this choice is found in Jordan, Jorgensen, and Kuipers (2000), who show that there are many artificial prices at the long end of the Treasury yield curve. Due to idiosyncratic variation in the short end of the term structure, which has been documented for bills, coupon securities, and STRIPS, we only use data with more than one year to maturity.

We fit both the standard CIR model and the constant drift version on the first Wednesday of the month. If either model does not converge on that day, we fit the model using either the Tuesday or Thursday adjacent to the first Wednesday of the month. We assume that the variation of the parameters is due to statistical variance. In this regard, our approach is similar to Brown and Dybvig (1986), and is a very common approach in fixed income literature.
To perform nonlinear regressions on each cross-section of data, we use the Marquardt method, which is a variation of the Gauss-Newton procedure. A separate regression is run for both models on each day. This then results in estimates of the $\kappa$, $\mu$, $\sigma$, and $r$ parameters for the CIR model and estimates of the $\mu$, $\sigma$, and $r$ parameters for the adapted model. This method also produces standard errors for each parameter estimate.

**IV. EMPIRICAL RESULTS**

Figure III illustrates the estimates of the asymptotic zero-coupon rates from each of the term structure models. We present results using the CIR model in Panel A and results using the constant-drift version of the CIR model in Panel B. In this figure, estimated asymptotic forward rates, which are identical to asymptotic zero-coupon rates, are illustrated. The estimations of both the discount function (STRIPS prices) and the term structure (STRIPS yields) are presented. Vacant areas occur when either the model did not converge or when it produced negative yields, which are economically meaningless. These estimation problems occur when the term structure is very flat, in which case the CIR model appears to be over-parameterized, meaning widely varying sets of parameter estimates fit the empirical term structure quite well and unique point estimates are therefore not obtainable.

[Figure III about here]
The results in Panel A show that during the first half of the 1990s the asymptotic rates are relatively stable between 7% and 9%. However, over the second half of the 1990s, rates fall substantially, reaching levels below 2%. The large section of missing estimates in 1998 is caused by the flatness of the term structure during that period. In general, the price fit results are very similar to the yield fit results.

In Panel B, results are displayed from our estimations of the adapted constant drift model. The estimates produced by this model appear to be very similar to the CIR results. For this simpler model there are no instances of convergence problems or negative yields. Consistent with results in Panel A, the asymptotic yield is relatively stable in the first half of the 1990s and then falls during the second half of the decade. Here, the price fit results almost exactly mirror the yield fit results.

To further explore the significance of these results, we calculate the actual one-year forward rates implied by the data for the last year of our data set (year 24 to year 25). These one-year rates at year 24 are then compared to year 24 forward rates implied by the constant drift model. These results are presented in Figure IV. For comparison, the asymptotic forward rates from the constant drift model are also displayed.

Figure IV about here
As shown in Figure IV, the behavior of the actual year 24 forward rates resembles the behavior of the predicted year 24 forward rates as well as the predicted infinite horizon forward rates. These results further emphasize the occurrence of falling long interest rates during the last half of the 1990s.

V. CONCLUSIONS

This paper empirically examines the theoretical implications of Dybvig, Ingersoll, and Ross (DIR, 1996) that long forward and zero-coupon rates can never fall. We use monthly U.S. Treasury STRIPS data, for the period of 1990 to 2000, to predict infinite horizon forward rates using a common one-factor term structure model and an adaptation of that model. Our results for this sample period show that, in contrast to DIR’s theoretical conclusions, implied long forward and zero-coupon rates fell substantially. During the last half of the 1990s, the asymptotic rates fell from a relatively stable level of 8 percent to around 5 percent on average. The period we examined experienced a decline in short-term interest rates. An examination of a period when short-term interest rates rose may indicate that long rates increased during that period. Whether long rates do increase during a period of rising short-term rates is irrelevant to our conclusion since, according to term structure theory, long rates should never fall during any period.

So, do long rates actually ever fall? Orthodox term structure theory suggests that they cannot; our results suggest that they did. Our findings therefore
remind us of the story of Galileo when he publicly recanted his heretical view that the earth was not the stationary center of the cosmos. As legend has it, under his breath, he said “Eppur si muove” (“Nevertheless, it moves”). To paraphrase Galileo, we conclude that although long forward rates should not fall, "Eppur cadono" ("Nevertheless, they fall").
References


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Table 1
(Yields are continuously compounded and computed from market bid quotations.)

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<th>Maturity</th>
<th>Factor 1</th>
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<td>0.3054</td>
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<td>day 180</td>
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Variance explained by the model

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Table 1. Maximum Likelihood Factor Analysis. The data are monthly bid prices for U.S. Treasury coupon STRIPS for January 1, 1990 through December 31, 2000. The table shows the percent of the variation in the STRIPS yield curve explained by the first and second factor.
Figure I. Term Structure. Presented is a graph of the term structure assuming interest rates in one year will be either 6 percent with probability $p$ or 8 percent with probability $(1 - p)$ and stay that way forever.
**Figure II. Convexity.** Shown are the percentage change in the price of a pure discount bond from a one basis point change in interest rates, i.e., the value of an ‘01.
Figure III. Asymptotic Zero-Coupon Rates. Presented are monthly estimates of the asymptotic zero-coupon rates for the period January 1, 1990 through December 31, 2000. Results using the CIR model are presented in Panel A, while results using the constant-drift version of the CIR model are in Panel B.
Figure IV. Estimated Forward Rates: 1990-2000. Shown are forward rate estimates for the period January 1, 1990 through December 31, 2000. Reported are the actual one-year forward rates implied by the data for year 24, the year 24 forward rates implied by the constant drift model, and the asymptotic forward rates from the constant drift model.