Value at Risk

Portfolio variance for 2 asset portfolio (total risk) = $w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B Cov(A,B)$

Portfolio variance for 2 asset portfolio (total risk) = $w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \sigma_A \sigma_B \rho_{A,B}$

Asset returns follow a normal distribution?

Expected return of a portfolio:

$E(R_{P,T}) = E(R_P) \times T$

Standard deviation of a portfolio:

$\sigma_{P,T} = \sigma_P \times \sqrt{T}$

Suppose a portfolio has an annual standard deviation of 30 percent. What is the monthly standard deviation? The weekly standard deviation? The two year standard deviation?

$\sigma_{Monthly} = .30 \times \sqrt{1/12} = 8.66\%$

$\sigma_{Weekly} = .30 \times \sqrt{1/52} = 0.58\%$

$\sigma_{2\text{-year}} = .30 \times \sqrt{2} = 42.43\%$

Note, the variance is multiplied by T. In the above example, the 2-year variance is:

$\sigma^2 = .30^2 = .09$
$\sigma^2_{2-year} = .09(2) = .18$
$\sigma_{2\text{-year}} = \sqrt{.18} = 42.43\%$

Value at risk:

$\text{Prob}[R_{P,T} \leq E(Rp) \times T - 2.326\sigma_P\sqrt{T}\,] = 1\%$
$\text{Prob}[R_{P,T} \leq E(Rp) \times T - 1.96\sigma_P\sqrt{T}\,] = 2.5\%$
$\text{Prob}[R_{P,T} \leq E(Rp) \times T - 1.645\sigma_P\sqrt{T}\,] = 5\%$

A portfolio has an annual return of 15 percent with an annual return standard deviation of 50 percent.

What loss level can we expect over a two-year investment horizon with a probability of .17?

> We assume a two-year expected return of 30 percent. The 2-year standard deviation is $.50\sqrt{2} = .7071$, or 70.71 percent. A loss probability of .17 corresponds to one standard deviation below the mean, so the answer to our question is .30 – .7071 = –.4071, or
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> $\text{Prob}(R_P \leq -40.71\%) = 17\%$

What loss level can we expect over the next six months with a probability of .17?

> The six-month expected return is half of 15 percent, or 7.5 percent. The six-month standard deviation is $.5\sqrt{1/2} = .3536$. So the answer to our question is .075 – .3536 = –.2786, or
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> $\text{Prob}(R_P \leq -27.86\%) = 17\%$

What is the expected loss over the next year with a 5 percent probability?

> $\text{Prob}[R_{P,1} \leq E(Rp) \times T - 1.645\sigma_P\sqrt{T}\,] = 5\%$
> $\text{Prob}[R_{P,1} \leq 15\% \times 1 - 1.645(.50)\sqrt{1}\,] = 5\%$
> $\text{Prob}[R_{P,1} \leq -67.25\%] = 5\%$

What is the expected loss over the month with a 1 percent probability?

> $\text{Prob}[R_{P,1/12} \leq E(Rp) \times T - 2.326\sigma_P\sqrt{T}\,] = 1\%$
> $\text{Prob}[R_{P,1/12} \leq 15\% \times 1/12 - 2.326(.50)\sqrt{1/12}\,] = 1\%$
> $\text{Prob}[R_{P,1/12} \leq -32.32\%] = 1\%$