

## Term Structure of Interest Rates

Term Structure versus Yield Curve

### Term Structure Theories

1) Market segmentation

2) Rational expectations

Notation:  $f_{1,1}$  = 1-year interest rate in one year  
 $f_{2,1}$  = 2-year investment in one year  
 $f_{1,2}$  = 1-year investment in two years

Suppose a 1-year T-bill has a YTM of 5% and a 2-year STRIPS has a YTM of 6%. What is the one year interest rate in one year?

*2-year investment:  $(1 + .06)(1 + .06) - 1 = .1236$  or 12.36%*  
 *$\$100$  today =  $\$112.36$  in 2 years*  
 *$\$100$  today =  $\$105$  in 1 year*  
 *$\$105(1 + f_{1,2}) = \$112.36$*   
 *$f_{1,2} = .0667$  or 6.67%*

Suppose a 1-year T-bill has a YTM of 5% and a 3-year STRIPS has a YTM of 6%. What is the two year interest rate in one year?

$$\begin{aligned}
 \text{3-year investment: } & (1 + .06)^3 = 1.19106 \\
 & (1 + .05)(1 + f_{2,1})^2 = 1.19106 \\
 & (1 + f_{2,1})^2 = 1.134300952 \\
 & f_{2,1} = .0650 \text{ or } 6.50\%
 \end{aligned}$$

Suppose a 2-year STRIPS has an interest rate of 6% and a 3-year STRIPS has an interest rate of 7%. What is the 1-year interest rate in 2 years?

$$\begin{aligned}
 (1 + .06)^2(1 + f_{1,2}) &= (1 + .07)^3 \\
 f_{1,2} &= [(1 + .07)^3 / (1 + .06)^2] - 1 \\
 f_{1,2} &= .0903 \text{ or } 9.03\%
 \end{aligned}$$

*Geometric average*

$$1 + R_G = [(1 + R_1)(1 + R_2)\dots(1 + R_n)]^{1/n}$$

3) Liquidity preference – *investors prefer liquidity – rational expectations with liquidity premium which results in long-term bonds having a greater liquidity premium*

U.S. Treasury STRIPS, Feb, 15, 2018

<b>Maturity</b>	<b>Price</b>
Feb 19	97.100
Feb 20	93.875
Feb 21	90.123

1) According to the pure expectations theory of interest rates, how much do you expect to pay for a one-year STRIPS on February 15, 2019? What is the corresponding implied forward rate? How does your answer compare to the current yield on a one-year STRIPS? What does this tell you about the relationship between implied forward rates, the shape of the zero coupon yield curve, and market expectations about future spot interest rates?

2) Suppose the term structure is set according to pure expectations and the maturity preference theory. To be specific, investors require no compensation for holding investments with a maturity of one year, but they demand a premium of .30 percent for holding investments with a maturity of two years. Given this information, how much would you pay for a one-year STRIPS on February 15, 2019? What is the corresponding implied forward rate? Compare your answer to the solutions you found in the previous problem. What does this tell you about the effect of a maturity premium on implied forward rates?

$$[1 + (.03185/2)]^4 = [1 + (.02965/2)]^2 (1 + f_{1,1} + .0030) ; f_{1,1} = 3.135\%$$

$$f_{1,1} = 100/(1.03135) = 96.960\%$$

*Intuitively, the maturity premium on 2-year investments makes the future 1-year STRIP more valuable; hence, the forward price is greater and the forward rate lower. Alternatively, verify that if the forward rate and 1-year spot rate stayed the same as before, the spot 2-year price would become 93.6711% of par and the corresponding yield would be 3.323%; i.e., the longer maturity investment would be less valuable.*

4) Modern theories

*Fisher effect:*

$$1 + R = (1 + r)(1 + h)$$

*Nominal interest rate = Real interest rate + Inflation premium + Interest rate risk  
Premium + Liquidity premium + Default risk premium*

<https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/Historic-Yield-Data-Visualization.aspx>

**Imputed or Inferred Term Structure**

Suppose we have a 1 year zero coupon with a YTM of 6% and a 2 year, 8% annual coupon bond with a YTM of 8%. Assuming annual compounding, what is the one year zero rate in one year?

*Price of coupon bond = \$965.29*

*\$80 in one year, \$1,080 in 2 years --- Both are zeroes*

*The PV of \$80 in one year at 6% is \$75.47, so the PV of \$1,080 in 2 years must be: \$965.29 – 75.47 = \$889.82*

*If \$889.82 is the PV and \$1,080 is the FV in 2 years, the 2 years interest rate must be 10.17%*

Suppose we have the following one year bonds (semiannual coupon rates):

8% coupon  
7% YTM  
Price = \$1,009.50

6% coupon  
7% YTM  
Price = \$990.50

What are the six month zero rates and the 1 year zero rates?

$$1,009.50 = \$40d_1 + \$1,040d_2$$

$$990.50 = \$30d_1 + \$1,030d_2$$

*First equation times 3, second equation times -4*

$$3,028.50 = \$120d_1 + \$3,120d_2$$

$$-3,962 = -\$120d_1 - \$4,120d_2$$

$$933.50 = 1,000d_2$$

$$d_2 = .9335$$

$$d_2 = 1 / (1 + R_2)^2 = .9335$$

$$1 = .9335(1 + R_2)^2$$

$$1 / .9335 = (1 + R_2)^2$$

$$\sqrt{1 / .9335} - 1 = R_2$$

$$R_2 = .035 \text{ or } 3.5\% \rightarrow 7\% \text{ APR}$$

$$1,009.50 = \$40d_1 + \$1,040(.9335)$$

$$38.66 = 40d_1$$

$$.9655 = d_1$$

$$D_1 = 1 / (1 + R_1) = .9655$$

$$.9655 + .9655R_1 = 1$$

$$R_1 = .03466 \text{ or } 3.466\% \rightarrow 6.93\% \text{ APR}$$