Multiple Choice – Circle the correct answer – 3 points each – 30 points total

1. A coupon bond pays annual interest, has a par value of $1,000, matures in 12 years, has a coupon rate of 8.7%, and has a yield to maturity of 7.9%. The current yield on this bond is ___________.
   A.  8.39%
   B.  8.43%
   C.  8.83%
   D.  8.56%
   E.  None of the above

2. An inverted yield curve implies that:
   A.  Long-term interest rates are lower than short-term interest rates.
   B.  Long-term interest rates are higher than short-term interest rates.
   C.  Long-term interest rates are the same as short-term interest rates.
   D.  Intermediate term interest rates are higher than either short- or long-term interest rates.
   E.  None of these is correct.

3. The duration of a bond is a function of the bond's
   A.  coupon rate.
   B.  yield to maturity.
   C.  time to maturity.
   D.  All of the above.
   E.  None of the above

4. Which of the following is not true?
   A.  Holding other things constant, the duration of a bond increases with time to maturity.
   B.  Given time to maturity, the duration of a zero-coupon decreases with yield to maturity.
   C.  Given time to maturity and yield to maturity, the duration of a bond is higher when the coupon rate is lower.
   D.  Duration is a better measure of price sensitivity to interest rate changes than is time to maturity.
   E.  Holding other factors constant, the interest-rate risk of a coupon bond is lower when the bond's yield to maturity is higher.
5. Which one of the following statements is true concerning the duration of a perpetuity?

A. The duration of 15% yield perpetuity that pays $100 annually is longer than that of a 15% yield perpetuity that pays $200 annually.
B. The duration of a 15% yield perpetuity that pays $100 annually is shorter than that of a 15% yield perpetuity that pays $200 annually.
C. The duration of a 15% yield perpetuity that pays $100 annually is equal to that of 15% yield perpetuity that pays $200 annually.
D. The duration of 15% yield perpetuity that pays $100 annually is longer than that of a 15% yield perpetuity that pays $200 annually, and the duration of a 15% yield perpetuity that pays $100 annually is shorter than that of a 15% yield perpetuity that pays $200 annually.
E. All of these are false.

6. Identify the bond that has the longest duration.

A. 20-year maturity with an 8% coupon.
B. 20-year maturity with a 12% coupon.
C. 20-year maturity with a 0% coupon.
D. 10-year maturity with a 15% coupon.
E. 12-year maturity with a 12% coupon.

7. Which of the following are true about the interest-rate sensitivity of bonds?

I) Bond prices and yields are inversely related.
II) Prices of long-term bonds tend to be more sensitive to interest rate changes than prices of short-term bonds.
III) Interest-rate risk is directly related to the bond's coupon rate.
IV) The sensitivity of a bond's price to a change in its yield to maturity is inversely related to the yield to maturity at which the bond is currently selling.

A. I and II
B. I and III
C. I, II, and IV
D. II, III, and IV
E. I, II, III, and IV

8. According to modern term structure theory, which of the following is not represented in the term structure of interest rates?

A. A liquidity premium.
B. A coupon rate premium.
C. An interest rate risk premium.
D. A default premium.
E. A real rate of interest.
9. A Treasury bill has a face value of $200,000, an asked yield of 2.12 percent, and matures in 28 days. What is the price of this bill?

   A. $199,397.19  
   B. $199,408.08  
   C. $199,531.76  
   D. $199,670.22  
   E. $199,717.08

10. A Treasury bill is quoted at a bank discount yield of 1.08 percent and has 12 days to maturity. What is the bond equivalent yield given that this is a leap year?

   A. 1.08 percent  
   B. 1.13 percent  
   C. 1.10 percent  
   D. 1.12 percent  
   E. 1.06 percent
Partial Credit Problems – Show All Work – 70 Total Points

Problem 1 (15 points) There is a bond with 23 years to maturity, a 6.2 percent coupon paid semiannually, and a YTM of 4.8 percent. What is the Macaulay duration, convexity, modified duration, effective duration, and effective convexity? What is the predicted bond price using Macaulay duration and convexity and effective duration and convexity for a 1 percent increase in interest rates? How do each of these compare to the actual new price?
Problem 2 (5 points) Identify and describe five securities that directly apply to the money market.
Problem 3 (5 points) List and briefly explain five of Malkiel’s six theorems regarding bond prices and interest rates.
Problem 4 (10 points) You are responsible for managing a corporate liability fund. The fund is expected to make the payments below each year for the next five years and then make no more payments.

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$70,000,000</td>
</tr>
<tr>
<td>2</td>
<td>$90,000,000</td>
</tr>
<tr>
<td>3</td>
<td>$80,000,000</td>
</tr>
<tr>
<td>4</td>
<td>$60,000,000</td>
</tr>
<tr>
<td>5</td>
<td>$30,000,000</td>
</tr>
</tbody>
</table>

Assume all cash flows will occur at the end of the year and the interest rate is 4 percent. You have two bond issues that can be used to immunize our portfolio. Bond Y is a 1.5-year maturity bond with a duration of 1.26 years. Bond Z is a 7-year, zero coupon bond. What market value of each bond is necessary to fully immunize the liability?
Problem 5 (10 points) You find the following spot rates:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>R₁</td>
<td>4.9%</td>
</tr>
<tr>
<td>R₂</td>
<td>5.7%</td>
</tr>
<tr>
<td>R₃</td>
<td>6.4%</td>
</tr>
<tr>
<td>R₄</td>
<td>7.1%</td>
</tr>
</tbody>
</table>

a. What are the one year, two year, and three year forward rates in one year?
b. What are the one year rates in two years and three years?
Problem 6 (10 points) The term structure for zero coupon bonds is shown below. What is the yield to maturity of a 3 year bond with a $2,000 par value and an annual coupon rate of 6 percent? Assume annual interest rates.

<table>
<thead>
<tr>
<th>Rate</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.8%</td>
</tr>
<tr>
<td>2</td>
<td>3.4%</td>
</tr>
<tr>
<td>3</td>
<td>3.9%</td>
</tr>
</tbody>
</table>
Problem 7 (8 points) The yield to maturity (YTM) on 1-year zero-coupon bonds is 5 percent and the YTM on 2-year zeros is 5.75 percent. The yield to maturity on 2-year-maturity coupon bonds with coupon rates of 12 percent is 5.8 percent. The coupon bond is strippable. What arbitrage opportunity is available? What is the profit on the activity? Assume that all interest rates and coupons are annual.
Problem 8 (7 points) The 6-month Treasury bill spot rate is 4 percent, and the 1-year Treasury bill spot rate is 5 percent. What is the implied 6-month forward rate for 6 months from now?
Multiple Choice
1. E  
2. A  
3. D  
4. B  
5. C  
6. C  
7. C  
8. B  
9. D  
10. C

\[ \text{BET} = \frac{365 \times .0201}{360 - 30 \times .0201} = 2.04 \text{ percent} \]

\[ \text{BET} = \frac{366 \times .0108}{360 - 12 \times .0108} = 1.10 \text{ percent} \]

Problem #1
\[ D = \frac{1 + y}{y} - \frac{(1 + y + T(c - y))}{c[(1 + y)^T - 1] + y} = \frac{1 + .024}{.024} - \frac{(1 + .024) + 46(.031 - .024)}{.031[(1 + .024)^{46} - 1] + .024} \]
\[ D = 42.6667 - \frac{1.346}{.085291} = 26.88541 \text{ half years or 13.4427 years} \]

\[ \text{CX} = 10^8 \left[ \frac{1,192.132035804 - 1,193.6976400098}{1,193.6976400098} + \frac{1,195.266125212 - 1,193.6976400098}{1,193.6976400098} \right] \]
\[ \text{CX} = 241.3359 \]

\[ \text{D}_{\text{Mod}} = \frac{D}{1 + R} = \frac{13.4427}{1 + .024} = 13.1276 \]

Assuming a 100 bp change in rates:
\[ V_0 = $1,193.70 \]
\[ V_- = $1,365.86 \]
\[ V_+ = $1,050.45 \]

\[ \text{D}_{\text{Eff}} = \frac{V_- - V_+}{2 \times V_0 \times \Delta y} = \frac{1,365.86 - 1,050.45}{2(1,193.70)(.01)} = 13.2116 \]

\[ \text{C}_{\text{Eff}} = \frac{V_+ + V_- - 2V_0}{2V_0(\Delta y)^2} = \frac{1,365.86 + 1,050.45 - 2(1,193.706)}{2(1,193.70)(.01)^2} = 121.1349 \]

New price @5.8%: Actual = $1,050.45

Macaulay duration:
\[ \delta P = P \{ -D \left( \frac{\Delta R}{1 + R} \right) + \frac{1}{2} CX(\Delta R)^2 \} \]

\[ \delta P = $1,193.70 \{-13.4427 \left( \frac{.01}{1 + .048/2} \right) + \frac{1}{2} (241.3359)(.01)^2 \} \]

\[ \delta P = -$142.30 \]

New price = $1,051.40

Effective duration:

\[ \delta P = P \{ -D \left( \frac{\Delta R}{1 + R} \right) + \frac{1}{2} CX(\Delta R)^2 \} \]

\[ \delta P = $1,193.70 \{-13.2116 \left( \frac{.01}{1 + .048/2} \right) + \frac{1}{2} (121.1349)(.01)^2 \} \]

\[ \delta P = -$146.78 \]

New price = $1,046.92

**Problem #2**

Discount rate: rate the Federal Reserve charges banks for overnight reserve loans.

Federal fund: rate banks charge other banks for overnight loans of $1 million or more.

Call money: rate banks charge brokerage houses for call money loans.

Commercial paper: rate on short-term, unsecured debt issued by large corporations.

Negotiable CDs: rate banks pay on CDs over $100,000.

Bankers' acceptance: rate on acceptances issued by the largest commercial banks.

Eurodollar: dollar-denominated outside the U.S.

LIBOR: rate paid by London banks for dollar deposits from other banks.

T-bill rate: rate paid on U.S. Treasury securities issued for one year or less.

Repurchase agreement: simultaneous sale and agreement to repurchase an asset, typically a T-bill for one day

Reverse repos: Buy T-bill today and sign contract to sell back, often tomorrow

**Problem #3**

1. There is an inverse relationship between interest rates and bond prices. If interest rates increase, bond prices decrease. If interest rates decrease, bond prices increase.

2. An increase in a bond’s yield to maturity results in a smaller price change than a price decrease from an increase in the yield to maturity of the same magnitude.

3. Interest rate risk is the risk that if interest rates increase, bond prices will decrease. All else the same, a longer term bond will have more interest rate risk than a shorter term bond.

4. The sensitivity of bond prices to changes in yields increases at a decreasing rate as maturity increases. In other words, interest rate risk is less than proportional to maturity.

5. All else the same, there is an inverse relationship between the coupon rate and interest rate risk. A bond with a lower coupon has more interest rate risk than a bond with a higher coupon.
6. The sensitivity of a bond’s price to a change in its yield is inversely related to the yield to maturity at which the bond is currently selling. A low YTM will result in a greater interest rate sensitivity while a high YTM will result in a lower interest rate sensitivity.

**Problem #4**
The duration of the liability is:

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash flow</th>
<th>PV</th>
<th>PV x T</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$70,000,000</td>
<td>$67,307,692.31</td>
<td>$67,307,692.31</td>
</tr>
<tr>
<td>2</td>
<td>$90,000,000</td>
<td>$83,210,059.17</td>
<td>$166,420,118.34</td>
</tr>
<tr>
<td>3</td>
<td>$80,000,000</td>
<td>$71,119,708.69</td>
<td>$213,359,126.08</td>
</tr>
<tr>
<td>4</td>
<td>$60,000,000</td>
<td>$51,288,251.46</td>
<td>$205,153,005.85</td>
</tr>
<tr>
<td>5</td>
<td>$30,000,000</td>
<td>$24,657,813.20</td>
<td>$123,289,066.01</td>
</tr>
<tr>
<td></td>
<td><strong>$297,583,524.84</strong></td>
<td><strong>$775,529,008.59</strong></td>
<td></td>
</tr>
</tbody>
</table>

D = $775,529,008.59 / $297,583,524.84 = 2.606089 years

\[ w_Y D_Y + w_Z D_Z = 2.606089 \text{ years} = w_Y (1.26) + (1 - w_Y)7 \]
\[ 5.74 w_Y = 3.1339 \]
\[ w_Y = .7655 \]
\[ w_Z = .2345 \]

Market value of Y = .7655($297,583,524.84) = $227,797,154.53
Market value of Z = .2345($297,583,524.84) = $69,788,370.61

**Problem #5**

a. \[ f_{1,1} = \left( \frac{1.0572}{1.049} \right)^{1/1} - 1 = 6.51\% \]
\[ f_{1,2} = \left( \frac{1.0643}{1.049} \right)^{1/2} - 1 = 7.16\% \]
\[ f_{1,3} = \left( \frac{1.0714}{1.049} \right)^{1/3} - 1 = 7.84\% \]

b. \[ f_{2,1} = 1.0643 / 1.0572 - 1 = 7.81\% \]
\[ f_{3,1} = 1.0714 / 1.0643 - 1 = 9.23\% \]

**Problem #6**

Present values:
$120 in one year @2.8\% = $116.73
$120 in two years @ 3.4\% = $112.24
$2,120 in three years @ 3.9\% = $1,890.12

Price = $2,119.09

YTM = 3.86\%
Problem #7
The price of the coupon bond, based on its yield to maturity, is $1,113.99. If the coupons were stripped and sold separately as zeros, then, based on the yield to maturity of zeros with maturities of one and two years, respectively, the coupon payments could be sold separately for:

$$\frac{120}{1.05} + \frac{1,120}{1.0575^2} = 1,115.80$$

The arbitrage strategy is to buy the bonds, strip the payments, and sell zeros with face values of $120 and $1,120, and respective maturities of one year and two years. The profit is $1,115.80 – 1,113.99 = $1.81 per bond.

Problem #8
The given rates are annual rates, but each period is a half-year. Therefore, the per-period spot rates are 2.5% on one-year bonds and 2% on six-month bonds. The semiannual forward rate is obtained by solving for \( f \) in the following equation:

$$1 + f = \frac{1.025^2}{1.02} = 1.030$$

This means that the forward rate is 0.030 = 3.0% semiannually, or 6.0% annually.