Put-Call Parity with Known Dividend
\[ C - P = S - (\text{Div})e^{-Rt} - Xe^{-Rt} \]

Put-Call Parity with Continuous Dividends
\[ P = C + Xe^{-Rt} - S_0e^{-yt} \]

Black-Scholes-Merton Model
\[ C_0 = S_0e^{-yt}N(d_1) - Xe^{-Rt}N(d_2) \]
\[ P_0 = Xe^{-Rt}N(-d_2) - S_0e^{-yt}N(-d_1) \]
\[ d_1 = \frac{ln(S_0/X) + (R - y + \sigma^2/2)t}{\sigma \sqrt{t}} \]
\[ d_2 = d_1 - \sigma \sqrt{t} \]

Delta of a call = \( e^{-yt}N(d_1) \)
Delta of a put = \( -e^{-yt}N(-d_1) \)

Eta of a call = \( e^{-yt}N(d_1)(S/C) \)
Eta of a put = \( -e^{-yt}N(-d_1)(S/P) < 0 \)

Vega = \( S_0e^{-yt}N'(d_1) \sqrt{t} \)
\[ N'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \]

Gamma = \( \frac{N'(d_1)e^{-yt}}{s_0 \sigma \sqrt{t}} \)

Call theta = \( -\frac{S_0N'(d_1)e^{yt}}{2\sqrt{t}} + yS_0N(d_1)e^{yt} - RXe^{-Rt}N(d_2) \)
Put theta = \( -\frac{S_0N'(d_1)e^{yt}}{2\sqrt{t}} - yS_0N(-d_1)e^{yt} + RXe^{Rt}N(-d_2) \)

Call rho = \( Xt e^{-Rt}N(d_2) \)
Put rho = \( -Xt e^{-Rt}N(-d_2) \)

Hedging with index options
Number of option contracts = \( \frac{\text{Portfolio beta} \times \text{Portfolio value}}{\text{Option delta} \times \text{option contract value}} \)

Binomial trees
\[ p^* = e^{\mu t - d} \]
\[ p = \frac{u-d}{u-d} \]
\[ u = e^{\sigma \sqrt{\Delta t}} \]
\[ d = e^{-\sigma \sqrt{\Delta t}} = 1/u \]
\[ f = e^{R\Delta t}[pf_u + (1-p)f_d] \]
Known Dividend
\( S^* = S_0 - (\text{Dividend})e^{-Rt} \)

Continuous dividend yield and binomial trees
\[
p = \frac{e^{(R - \theta) \Delta t} - d}{u - d},\nonumber
\]
\[
u = e^{\sigma \sqrt{\Delta t}}\nonumber
\]
\[
d = e^{-\sigma \sqrt{\Delta t}} = 1/ u\nonumber
\]

Options on futures
\[
u = e^{\sigma \sqrt{\Delta t}}\nonumber
\]
\[
d = e^{-\sigma \sqrt{\Delta t}} = 1/ u\nonumber
\]
\[
p = \frac{1 - d}{u - d}\nonumber
\]

Money Markets
Price = Face value \( \left(1 - \frac{\text{Days}}{360} \times R_{BD}\right) \)
\[
R_{BD} = \frac{\text{Par-Price} \times 360}{\text{Par} \times n}\nonumber
\]
\[
R_{BEY} = \frac{\text{Par-Price} \times 365}{\text{Price} \times n}\nonumber
\]
\[
R_{BEY} = \frac{365 \times R_{BD}}{360 \times (R_{BD} \times n)}\nonumber
\]

Equivalent taxable yield = \( \frac{\text{Tax-exempt yield}}{1 - \text{Marginal tax rate}} \)

Critical tax rate = \( 1 - \frac{R_M}{R} \)

Accrued interest
30/360
If \( D_1 = 31 \), change to 30
If \( D_2 = 31 \) and \( D_1 = 30 \) or 31, change \( D_2 \) to 30, otherwise leave \( D_2 \) at 31
\# of days
\[
(Y_2 - Y_1) \times 360 + (M_2 - M_1) \times 30 + (D_2 - D_1)\nonumber
\]
30E/360 – Assumes a 30-day month
If \( D_1 = 31 \), change to 30
If \( D_2 = 31 \) Change to 30
\# of days
\[
(Y_2 - Y_1) \times 360 + (M_2 - M_1) \times 30 + (D_2 - D_1)\nonumber
\]

\( w = \frac{\# \text{ of days between settlement and next coupon payments}}{\# \text{ of days in coupon period}} \)
Accrued interest = \( C \left( \frac{\text{# of days since last coupon}}{\text{# of days in period}} \right) \)

**Duration and Convexity**

\[
\frac{\partial P}{P} = -D \left( \frac{\partial R}{1 + R} \right)
\]

\[
\frac{\partial P}{P} = -D \left( \frac{\partial R}{1 + (R/2)} \right)
\]

\[
D = \frac{\sum \text{DCF} \times t}{\sum \text{DCF} \times (\text{price})}
\]

\[
D = \frac{1 + y}{y} \cdot \frac{(1 + y) + T(c - y)}{c[(1 + y)^T - 1] + y}
\]

Duration of a perpetuity is: \( \frac{1 + y}{y} \)

Duration for a level annuity is: \( \frac{1 + y}{y} \cdot \frac{T}{(1 + y)^T - 1} \)

\[
\frac{\partial P}{P} = P \times [(-D) \times \left( \frac{\partial R}{1 + R} \right)]
\]

\[
\frac{\partial P}{P} = -D \left[ \frac{\Delta R}{1 + R} \right] + \frac{1}{2} \text{CX}(\Delta R)^2
\]

\( \text{CX} = \text{convexity} = \text{Scaling factor} \) [capital loss from one basis point rise in R] + [capital gain from one basis point drop in R]

\[
D_M = \frac{D}{1 + y}
\]

\( \% \Delta \text{ in bond price} = -D_M(\Delta R) \)

\[
D_E = \frac{V_- - V_+}{2V_0(\Delta R)}
\]

\( V_0 = \text{initial price} \)

\( V_- = \text{price if YTM decreases by } R \)

\( V_+ = \text{price if YTM increases by } R \)

\[ V_- + V_+ - 2V_0 \]

\[
\text{CX}_E = \frac{V_- + V_+ - 2V_0}{2V_0(\Delta R)^2}
\]
Futures
\[ F_T = S(1+ R - d)^T \]

Stock hedging with futures
\[ \frac{\text{# of contracts}}{V_F} = \frac{\beta_P \times V_P}{V_F} \]

Bond hedging with futures
\[ \frac{\text{# of contracts}}{V_F} = \frac{D_P \times V_P}{D_F \times V_F} \]

Cross Hedging
\[ h = \rho_{S,F} \left( \frac{\sigma_s}{\sigma_F} \right) \]

Value at Risk

Portfolio variance for 2 asset portfolio (total risk) = \[ w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_Aw_BCov(A,B) \]

Portfolio variance for 2 asset portfolio (total risk) = \[ w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_Aw_B\sigma_A\sigma_B\rho_{A,B} \]

\[ E(R_{P,T}) = E(R_p) \times T \]
\[ \sigma_{P,T} = \sigma_p \times \sqrt{T} \]

\[ \text{Prob}[R_{P,T} \leq E(R_p) \times T - 2.326\sigma_p\sqrt{T}] = 1\% \]
\[ \text{Prob}[R_{P,T} \leq E(R_p) \times T - 1.96\sigma_p\sqrt{T}] = 2.5\% \]
\[ \text{Prob}[R_{P,T} \leq E(R_p) \times T - 1.645\sigma_p\sqrt{T}] = 5\% \]