Value at Risk

Portfolio variance for 2 asset portfolio (total risk) = \( w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_Aw_B \text{Cov}(A,B) \)

Portfolio variance for 2 asset portfolio (total risk) = \( w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2w_Aw_B \sigma_A \sigma_B \rho_{A,B} \)

Asset returns follow a normal distribution?

Expected return of a portfolio:
\[
E(R_{P,T}) = E(R_P) \times T
\]

Standard deviation of a portfolio:
\[
\sigma_{P,T} = \sigma_P \times \sqrt{T}
\]

Suppose a portfolio has an annual standard deviation of 30 percent. What is the monthly standard deviation? The weekly standard deviation? The two year standard deviation?

\[
\sigma_{\text{Monthly}} = .30 \times \sqrt{1/12} = 8.66\%
\]
\[
\sigma_{\text{Weekly}} = .30 \times \sqrt{1/52} = 0.58\%
\]
\[
\sigma_{\text{2-year}} = .30 \times \sqrt{2} = 42.43\%
\]

Note, the variance is multiplied by T. In the above example, the 2-year variance is:
\[
\sigma^2 = .30^2 = .09
\]
\[
\sigma_{\text{2-year}}^2 = .09(2) = .18
\]
\[
\sigma_{\text{2-year}} = \sqrt{.18} = 42.43\%
\]
Value at risk:

\[
\text{Prob} [ R_{p,T} \leq E(R_p) \times T - 2.326 \sigma_p \sqrt{T} ] = 1% \\
\text{Prob} [ R_{p,T} \leq E(R_p) \times T - 1.96 \sigma_p \sqrt{T} ] = 2.5% \\
\text{Prob} [ R_{p,T} \leq E(R_p) \times T - 1.645 \sigma_p \sqrt{T} ] = 5% \\
\]

A portfolio has an annual return of 15 percent with an annual return standard deviation of 50 percent.

What loss level can we expect over a two-year investment horizon with a probability of .17?

We assume a two-year expected return of 30 percent. The 2-year standard deviation is $.50 \sqrt{2} = .7071$, or 70.71 percent. A loss probability of .17 corresponds to one standard deviation below the mean, so the answer to our question is $0.30 - 0.7071 = -0.4071$, or

\[
\text{Prob}(R_p \leq -40.71\%) = 17\%
\]

What loss level can we expect over the next six months with a probability of .17?

The six-month expected return is half of 15 percent, or 7.5 percent. The six-month standard deviation is $0.5 \sqrt{1/2} = 0.3536$. So the answer to our question is $0.075 - 0.3536 = -0.2786$, or

\[
\text{Prob}(R_p \leq -27.86\%) = 17\%
\]

What is the expected loss over the next year with a 5 percent probability?

\[
\text{Prob}[R_{p,1} \leq E(R_p) \times T - 1.645 \sigma_p \sqrt{T} ] = 5\% \\
\text{Prob}[R_{p,1} \leq 15\% \times 1 - 1.645(0.5)\sqrt{1} ] = 5\% \\
\text{Prob}[R_{p,1} \leq -67.25\%] = 5\%
\]

What is the expected loss over the month with a 1 percent probability?

\[
\text{Prob}[R_{p,1/12} \leq E(R_p) \times T - 2.326 \sigma_p \sqrt{T} ] = 1\% \\
\text{Prob}[R_{p,1/12} \leq 15\% \times 1/12 - 2.326(0.5)\sqrt{1/12} ] = 1\% \\
\text{Prob}[R_{p,1/12} \leq -32.32\%] = 1\%
\]